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Posted Date: 2 June 2026

doi: 10.20944/preprints202606.0079.v1

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Article

Neutrosophic Refined Power Mean Operator and Its Application for MADM Problem Based on Cross Entropy Measure

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Abstract

In real-world decision-making, constructing mathematical models is often difficult because the data are incomplete, uncertain, or even contradictory. The neutrosophic refined set provides a robust and flexible approach for effectively handling and representing these types of uncertainties. Various studies have highlighted its significant applications in decision making.In this study, a power mean operator is introduced to aggregate multiple Neutrosophic Refined Sets (NRSs) into a Single-Valued Neutrosophic Set (SVNs). The core mathematical properties of the newly introduced neutrosophic refined power mean operator are established. Moreover, two categories of neutrosophic refined cross-entropy measures are presented: one adapted from the SVNs-cross-entropy measure, and the other specifically formulated for neutrosophic refined sets. By employing the defined measures, an innovative decision making stratgegy is developed under the neutrosophic refined set environment. To demonstrate the effectiveness and practical relevance of the grounded strategy a numerical example based on the selection of an educational stream is solved.

Keywords: neutrosophic set; neutrosophic refined sets; MADM; neutrosophic refined power mean operator; cross entropy measure

1. Introduction

Multi-Attribute Decision Making (MADM) denotes a decision process in which the optimal alternative is determined from a collection of feasible options considering multiple, potentially conflicting, criteria. Because decision-making information is frequently uncertain and incomplete, selecting the most suitable alternative becomes a challenging task. Zadeh was the first to formulate the Fuzzy Set (FS) [1], incorporating a degree of membership for all elements. Since then, numerous researchers have applied fuzzy set theory to various decision-making problems [2–4]. Later, Krassimir Atanassov [5] extended the concept of FS by introducing the Intuitionistic Fuzzy Set (IFS), which incorporates an independent degree of non-membership. IFSs have also been utilized in a wide range of decision-making scenarios [6,7]. Despite their broad applicability, FSs and IFSs are insufficient for modeling situations where the indeterminacy function (I-function) has no dependence on either the membership function (T-function) or the non-membership function (F-function). In real-world decision-making processes, indeterminacy inherently arises due to incomplete and imprecise information. To address this limitation, Smarandache [8] proposed the Neutrosophic Set (NS), in which three mutually independent functions T-function, I-function, and F-function are defined over the interval $]^{-}0, 1^{+}[$. Subsequently, wang et al. [9] introduced the Single Valued NS (SVNS) as a particular case of the NS framework, where each membership component assumes values from $[0, 1]$.

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Over time, this concept has proven to be effective when applied in numerous fields and practical contexts [10–16]. Later, the neutrosophic framework was expanded to include interval neutrosophic sets (INS) [17], in which uncertainty is described using interval-based T-, I-, and F- function. Several studies have explored applications of INSs in different domains [18–24]. Later, the neutrosophic framework was expanded to include INS [17], in which uncertainty is described using interval-based components of T-, I-, and F-function. Several studies have explored the applications of INSs in different domains [18–24].

Motivation of the Study

In particular cases, a mathematical framework that can represent repeated occurrences of elements is needed, such as the symptoms of a patient measured over different time intervals. To deal with this kind of situation, we use the concept [25]. The concept of a multiset can be found in [26]. Later, several researchers have discussed more properties and applications of multiset [27–29]. The concept of fuzzy bags was formally defined by Baowen et al. [30] in 1998. Miyamoto [31] defined the concepts of multisets in fuzzy environment in 2001. Later, Rocacher [32] applied fuzzy bags to build up flexible query. Shinoj and Sunil [33] developed Intuitionistic Fuzzy mMltisets (IFMs) as an extension of fuzzy multisets and proposed various associated operations. In [33] Shinoj and Sunil employed the IFMs in medical diagnosis problem. In 2013, Smarandache [34] introduced the concept of a refined NS (RNS) by further developing the components of NS.

Subsequently, numerous researchers have applied NRSs to MADM problems using various approaches [35–40]. Cross entropy measure is a popular divergence measure in recent literature that calculates the divergence of any variable from other variables. The combination of cross entropy measure techniques with several decision-making environments by a lot of researchers are stated in Table 1.

Table 1. Cross entropy measure literature review other than NRSs environment.

Papers	Applied Techniques	Environment
Shang and Jiang [41]	Cross Entropy Measure (CEM)	FS
Vlachos and Sergiadis [42]	,,	IFS
Ye [43]	,,	IFS
Maheshwari and Srivastava [44]	,,	IFS
Zhang et al. [45]	,,	Interval-IFS
Ye [46]	,,	interval-IFS
Ye [47]	,,	SVNSs
Wu et al. [50]	,,	SVNSs
Ye [48]	,,	SVNSs
Pramanik et al. [11]	,,	SVNSs
Tian et al. [49]	,,	INSs
Sahin [18]	,,	INSs
Dalapati et al. [24]	,,	INSs
Pramanik et al. [51]	,,	NCSs

Several researchers discussed more excellent results based on the fuzzy multisets [52–54] and intuitionistic fuzzy multisets [55–61].

However, NRS is a mathematical tool to express an element by the repeated membership, indeterminacy, and falsity degrees. Table 2 shows that few studies have been performed in the environment of NRSs. However, no study has been reported in the literature so far with cross entropy measure in NRS environment despite significant advantages of cross entropy measure in the decision-making field. Motivated by these innovative ideas, we formulated a cross-entropy measure within the NRS environment and introduced a MADM approach applicable to the educational field.

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Table 2. NRSs decision making process literature review.

Papers	Applied Techniques	Environment
Broumi and Deli [35]	Correlation measure	NRSs
Ye and Ye [36]	Dice similarity measure	„
Karaaslan [37]	Correlation coefficient measure	„
Mondal and Pramanik [38]	Cotangent similarity measure	„
Pramanik et al. [39,40]	Decision making strategy	„
Ye [62]	Arctangent similarity measures	„
Tan [63]	New entropy measure	„
Fan [64]	Correlation coefficients	„
Karaaslan [65]	Similarity measures	„

Contribution of the Study

The primary objective of this research is to develop a methodology to identify the most suitable option among a set of feasible alternatives. To accomplish this objective within a decision-making framework, an aggregation operator is first formulated in general, and its crucial properties are rigorously established. Furthermore, a sensitivity analysis is performed to show the stability and effectiveness of the grounded operator.

The principal contributions of this research are summarized as:

- A novel concept of the power mean operator is introduced within the neutrosophic refined set (NRS) framework.
- We systematically proved its essential mathematical properties.
- Two distinct formulations of the CEM are proposed in the NRS Setting to quantify the degree of dissimilarity between two NRSs.
- An MADM framework based on the proposed CEM is introduced to address issues related to DM-problems involving uncertainty and imprecision in an efficient manner.
- Solve a real-world MADM problem using the proposed method to demonstrate its practical applicability in the current context.
- A sensitivity analysis isperformed to explore the effect of varying the parameter q on the decision-making outcomes of the operator.

Structure of This Study

The general structure of the remaining content is presented as follows. Section-2 covers the foundational concepts of NS, SVNS, and NRS, along with basic operations on NRS. This section also introduces the neutrosophic refined power mean operator, establishes its key properties, and discusses several special cases. Section 3 presents two types of NRS-based CEM and examines their main properties. Section 4 introduces an MADM approach utilizing the power mean operator to solve MADM g problems within the NRS framework. Section 5 presents a numerical illustration to assess the effectiveness of the developed approach, while Section 6 summarizes the study and outlines the concluding observations and future research directions.

2. Preliminaries

This section briefly discusses SVNS, and NRSs, along with a description of their fundamental set-theoretic operations.

Definition 1: SVNS [9]

Let $\mathfrak{U}$ represent a discourse universe that consists of elements (or objects), where each element in $\mathfrak{U}$ is denoted by $\sqcap$, i.e., $\sqcap \in \mathfrak{U}$. An SVNS $\mathcal{M}$ defined in $\mathfrak{U}$ is characterized by the T-function $\mathfrak{T}_M(\sqcap)$, the I-function $\mathfrak{I}_M(\sqcap)$, and the F-function $\mathfrak{F}_M(\sqcap)$. Each of these functions maps the elements of $\mathfrak{U}$ into the interval $[0,1]$, that is,

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$$\mathfrak{T}_M(\sqcap), \mathfrak{I}_M(\sqcap), \mathfrak{F}_M(\sqcap) : \mathfrak{U} \to [0,1].$$

Consequently, SVNS $\mathcal{M}$ can be represented as

$$\mathcal{M} = \{\langle \sqcap, (\mathfrak{T}_M(\sqcap), \mathfrak{I}_M(\sqcap), \mathfrak{F}_M(\sqcap))\rangle : \sqcap \in \mathfrak{U}\}.$$

Since $\mathfrak{T}_M(\sqcap)$, $\mathfrak{I}_M(\sqcap)$, and $\mathfrak{F}_M(\sqcap)$ take values in $[0,1]$, and their sum lies:

$$0 \leq \mathfrak{T}_M(\sqcap) + \mathfrak{I}_M(\sqcap) + \mathfrak{F}_M(\sqcap) \leq 3.$$

For simplicity, a SVNS can be denoted as $\mathcal{M} = \langle \mathfrak{T}, \mathfrak{I}, \mathfrak{F}\rangle$, which is referred to as a *single-valued neutrosophic number* (SVNN). Symbolically, it is represented in the form of an SVNN.

Example 1: Consider an SVNS $\mathcal{M}$ defined in the universe $\mathfrak{U}$. Then, $\mathcal{M}$ can be represented as

$$\mathcal{M} = \{\langle \sqcap, (0.8, 0.3, 0.4)\rangle : \sqcap \in \mathfrak{U}\}.$$

Consequently, the corresponding SVNN is expressed as

$$\mathcal{M} = \langle 0.8, 0.3, 0.4\rangle.$$

Definition 2:NRS [34]

Assume that $\mathfrak{U}$ is a non-empty universe with elements denoted by $\sqcap$. An NRS defined in $\mathfrak{U}$ is represented by $\mathcal{N}$ and expressed as

$$\mathcal{N} = \Big\{ \Big\langle \sqcap, (\mathfrak{T}_N^1(\sqcap), \mathfrak{T}_N^2(\sqcap), \ldots, \mathfrak{T}_N^P(\sqcap)), (\mathfrak{I}_N^1(\sqcap), \mathfrak{I}_N^2(\sqcap), \ldots, \mathfrak{I}_N^P(\sqcap)), (\mathfrak{F}_N^1(\sqcap), \mathfrak{F}_N^2(\sqcap), \ldots, \mathfrak{F}_N^P(\sqcap))\Big\rangle : \sqcap \in \mathfrak{U}\Big\}.$$

Each of the sequences $\mathfrak{T}_N^1(\sqcap), \mathfrak{T}_N^2(\sqcap), \ldots, \mathfrak{T}_N^P(\sqcap)$, $\mathfrak{I}_N^1(\sqcap), \mathfrak{I}_N^2(\sqcap), \ldots, \mathfrak{I}_N^P(\sqcap)$, and $\mathfrak{F}_N^1(\sqcap), \mathfrak{F}_N^2(\sqcap), \ldots, \mathfrak{F}_N^P(\sqcap)$ is a mapping from $\mathfrak{U}$ to $[0,1]$.

These sequences represent the T-function, the I-function, and the F-function, respectively.

Example 2: Assume that $\mathfrak{U} = \{\sqcap_1, \sqcap_2\}$ is a universe and consider $\mathcal{N}$ as a NRS defined in $\mathfrak{U}$. Then, $\mathcal{N}$ can be written as

$$\mathcal{N} = \left\{ \begin{array}{l} \langle \sqcap_1,\ (0.7, 0.5, 0.8),\ (0.2, 0.3, 0.1),\ (0.3, 0.4, 0.3)\rangle, \\ \langle \sqcap_2,\ (0.6, 0.6, 0.5),\ (0.4, 0.3, 0.3),\ (0.5, 0.4, 0.2)\rangle \end{array} \right\}.$$

The operations of NRSs described in [34] are presented below:

1. **Inclusion of NRSs**
   Let $\mathcal{N}_1$ and $\mathcal{N}_2$ be two NRSs defined in a universe $\mathfrak{U}$. The inclusion relation $\mathcal{N}_1 \subseteq \mathcal{N}_2$ holds if and only if, for every $\sqcap \in \mathfrak{U}$ and each refinement index $i = 1, 2, \ldots, p$, the following conditions are satisfied:
   $$\mathfrak{T}^i_{\mathcal{N}_1}(\sqcap) \leq \mathfrak{T}^i_{\mathcal{N}_2}(\sqcap), \quad \mathfrak{I}^i_{\mathcal{N}_1}(\sqcap) \geq \mathfrak{I}^i_{\mathcal{N}_2}(\sqcap), \quad \mathfrak{F}^i_{\mathcal{N}_1}(\sqcap) \geq \mathfrak{F}^i_{\mathcal{N}_2}(\sqcap).$$

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Example 3:
Consider two NRSs $\mathcal{N}_1$ and $\mathcal{N}_2$ on $\mathfrak{U}$ defined as:

$$\mathcal{N}_1 = \Big\{\langle \sqcap_1, (0.7, 0.5, 0.7), (0.2, 0.3, 0.1), (0.4, 0.5, 0.6)\rangle, \\ \langle \sqcap_2, (0.6, 0.6, 0.5), (0.7, 0.4, 0.5), (0.6, 0.7, 0.3)\rangle\Big\},$$

$$\mathcal{N}_2 = \Big\{\langle \sqcap_1, (0.8, 0.5, 0.9), (0.1, 0.2, 0.1), (0.3, 0.4, 0.3)\rangle, \\ \langle \sqcap_2, (0.6, 0.7, 0.8), (0.4, 0.3, 0.2), (0.5, 0.4, 0.2)\rangle\Big\}.$$

From the inclusion condition above, it follows that $\mathcal{N}_1 \subseteq \mathcal{N}_2$.

2. **Equality of Two NRSs**
Two NRSs $\mathcal{N}_1$ and $\mathcal{N}_2$ in $\mathfrak{U}$ are said to be equal, denoted by $\mathcal{N}_1 = \mathcal{N}_2$, if and only if for every $\sqcap \in \mathfrak{U}$ and each $i = 1, 2, \ldots, p$,

$$\mathfrak{T}^i_{\mathcal{N}_1}(\sqcap) = \mathfrak{T}^i_{\mathcal{N}_2}(\sqcap), \quad \mathfrak{I}^i_{\mathcal{N}_1}(\sqcap) = \mathfrak{I}^i_{\mathcal{N}_2}(\sqcap), \quad \mathfrak{F}^i_{\mathcal{N}_1}(\sqcap) = \mathfrak{F}^i_{\mathcal{N}_2}(\sqcap).$$

3. **Complement of an NRSs**
Let $\mathcal{N}$ be an NRS defined on the universe $\mathcal{U}$. The complement of $\mathcal{N}$, denoted by $\mathcal{N}^c$, is defined by:

$$\mathcal{N}^c = \Big\{\langle \sqcap, 1 - \mathfrak{T}^i_{\mathcal{N}}(\sqcap), 1 - \mathfrak{I}^i_{\mathcal{N}}(\sqcap), 1 - \mathfrak{F}^i_{\mathcal{N}}(\sqcap)\rangle \;\big|\; \sqcap \in \mathfrak{U},\ i = 1, 2, \ldots, p\Big\}.$$

Example 4: Let an NRS $\mathcal{N}_1$ on $\mathfrak{U}$ be expressed as:

$$\mathcal{N}_1 = \Big\{\langle \sqcap_1, (0.7, 0.5, 0.7), (0.2, 0.3, 0.1), (0.4, 0.5, 0.6)\rangle, \\ \langle \sqcap_2, (0.6, 0.6, 0.5), (0.7, 0.4, 0.5), (0.6, 0.7, 0.3)\rangle\Big\}.$$

Then, its complement $\mathcal{N}_1^c$ is obtained as:

$$\mathcal{N}_1^c = \Big\{\langle \sqcap_1, (0.3, 0.5, 0.3), (0.8, 0.7, 0.9), (0.6, 0.5, 0.4)\rangle, \\ \langle \sqcap_2, (0.4, 0.4, 0.5), (0.3, 0.6, 0.5), (0.4, 0.3, 0.7)\rangle\Big\}.$$

4. **Union of two NRSs**
Let $\mathcal{N}_1$ and $\mathcal{N}_2$ be two NRSs on the universe $\mathfrak{U}$. They can be written as

$$\mathcal{N}_1 = \Big\{\Big\langle \sqcap, (\mathfrak{T}^1_{N_1}(\sqcap), \mathfrak{T}^2_{N_1}(\sqcap), \ldots, \mathfrak{T}^p_{N_1}(\sqcap)), \\ (\mathfrak{I}^1_{N_1}(\sqcap), \mathfrak{I}^2_{N_1}(\sqcap), \ldots, \mathfrak{I}^p_{N_1}(\sqcap)), \\ (\mathfrak{F}^1_{N_1}(\sqcap), \mathfrak{F}^2_{N_1}(\sqcap), \ldots, \mathfrak{F}^p_{N_1}(\sqcap))\Big\rangle : \sqcap \in \mathfrak{U}\Big\},$$

and

$$\mathcal{N}_2 = \Big\{\Big\langle \sqcap, (\mathfrak{T}^1_{N_2}(\sqcap), \mathfrak{T}^2_{N_2}(\sqcap), \ldots, \mathfrak{T}^p_{N_2}(\sqcap)), \\ (\mathfrak{I}^1_{N_2}(\sqcap), \mathfrak{I}^2_{N_2}(\sqcap), \ldots, \mathfrak{I}^p_{N_2}(\sqcap)), \\ (\mathfrak{F}^1_{N_2}(\sqcap), \mathfrak{F}^2_{N_2}(\sqcap), \ldots, \mathfrak{F}^p_{N_2}(\sqcap))\Big\rangle : \sqcap \in \mathfrak{U}\Big\}.$$

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The union $\mathcal{N}_3 = \mathcal{N}_1 \cup \mathcal{N}_2$ is defined by

$$\begin{aligned} \mathfrak{T}^i_{N_3}(\sqcap) &= \max\{\mathfrak{T}^i_{N_1}(\sqcap), \mathfrak{T}^i_{N_2}(\sqcap)\}, \\ \mathfrak{I}^i_{N_3}(\sqcap) &= \min\{\mathfrak{I}^i_{N_1}(\sqcap), \mathfrak{I}^i_{N_2}(\sqcap)\}, \qquad \forall \sqcap \in \mathfrak{U},\ i = 1, \ldots, p. \\ \mathfrak{F}^i_{N_3}(\sqcap) &= \min\{\mathfrak{F}^i_{N_1}(\sqcap), \mathfrak{F}^i_{N_2}(\sqcap)\}, \end{aligned}$$

Example 5: Let us

$$\begin{aligned} \mathcal{N}_1 &= \{\langle (0.7, 0.5, 0.7), (0.2, 0.3, 0.1), (0.4, 0.5, 0.6)\rangle\}, \\ \mathcal{N}_2 &= \{\langle (0.8, 0.5, 0.9), (0.1, 0.2, 0.1), (0.3, 0.4, 0.3)\rangle\}. \end{aligned}$$

Then

$$\mathcal{N}_1 \cup \mathcal{N}_2 = \{\langle (0.8, 0.5, 0.9), (0.1, 0.2, 0.1), (0.3, 0.4, 0.3)\rangle\}.$$

5. **Intersection of two NRSs**

Let $\mathcal{N}_1$ and $\mathcal{N}_2$ be two NRSs on $\mathfrak{U}$. Their intersection $\mathcal{N}_4 = \mathcal{N}_1 \cap \mathcal{N}_2$ is defined by

$$\begin{aligned} \mathfrak{T}^i_{N_4}(\sqcap) &= \min\{\mathfrak{T}^i_{N_1}(\sqcap), \mathfrak{T}^i_{N_2}(\sqcap)\}, \\ \mathfrak{I}^i_{N_4}(\sqcap) &= \max\{\mathfrak{I}^i_{N_1}(\sqcap), \mathfrak{I}^i_{N_2}(\sqcap)\}, \qquad \forall \sqcap \in \mathfrak{U},\ i = 1, \ldots, p. \\ \mathfrak{F}^i_{N_4}(\sqcap) &= \max\{\mathfrak{F}^i_{N_1}(\sqcap), \mathfrak{F}^i_{N_2}(\sqcap)\}. \end{aligned}$$

2.1. Some Operation of NRSs [34]

Assume that $\mathcal{N}_1$ and $\mathcal{N}_2$ are two NRss defined in the universe $\mathfrak{U}$. For every $\sqcap \in \mathfrak{U}$ and each $i = 1, 2, \ldots, p$, the operations are described as follows:

1. **Additive operation**

$$\mathcal{N}_1 \oplus \mathcal{N}_2 = \Big\langle \mathfrak{T}^i_{\mathcal{N}_1}(\sqcap) + \mathfrak{T}^i_{\mathcal{N}_2}(\sqcap) - \mathfrak{T}^i_{\mathcal{N}_1}(\sqcap)\,\mathfrak{T}^i_{\mathcal{N}_2}(\sqcap), \mathfrak{I}^i_{\mathcal{N}_1}(\sqcap)\,\mathfrak{I}^i_{\mathcal{N}_2}(\sqcap),\ \mathfrak{F}^i_{\mathcal{N}_1}(\sqcap)\,\mathfrak{F}^i_{\mathcal{N}_2}(\sqcap)\Big\rangle.$$

2. **Multiplicative operation**

$$\mathcal{N}_1 \otimes \mathcal{N}_2 = \Big\langle \mathfrak{T}^i_{\mathcal{N}_1}(\sqcap)\,\mathfrak{T}^i_{\mathcal{N}_2}(\sqcap),\ \mathfrak{I}^i_{\mathcal{N}_1}(\sqcap) + \mathfrak{I}^i_{\mathcal{N}_2}(\sqcap) - \mathfrak{I}^i_{\mathcal{N}_1}(\sqcap)\,\mathfrak{I}^i_{\mathcal{N}_2}(\sqcap), \mathfrak{F}^i_{\mathcal{N}_1}(\sqcap) + \mathfrak{F}^i_{\mathcal{N}_2}(\sqcap) - \mathfrak{F}^i_{\mathcal{N}_1}(\sqcap)\,\mathfrak{F}^i_{\mathcal{N}_2}(\sqcap)\Big\rangle.$$

3. **Scalar multiplication**

$$\lambda\mathcal{N}_1 = \Big\{1 - \big(1 - \mathfrak{T}^i_{\mathcal{N}_1}(\sqcap)\big)^\lambda,\ \big(\mathfrak{I}^i_{\mathcal{N}_1}(\sqcap)\big)^\lambda,\ \big(\mathfrak{F}^i_{\mathcal{N}_1}(\sqcap)\big)^\lambda \ \Big|\ \lambda > 0\Big\}.$$

4. **Scalar exponentiation**

$$\mathcal{N}_1^\lambda = \Big\{\big(\mathfrak{T}^i_{\mathcal{N}_1}(\sqcap)\big)^\lambda,\ 1 - \big(1 - \mathfrak{I}^i_{\mathcal{N}_1}(\sqcap)\big)^\lambda,\ 1 - \big(1 - \mathfrak{F}^i_{\mathcal{N}_1}(\sqcap)\big)^\lambda \ \Big|\ \lambda > 0\Big\}, \quad (i = 1, 2, \ldots, p).$$

Definition 3: Neutrosophic Refined Power Mean Operator

Let $\mathcal{NRS}(\mathfrak{U})$ denote the family of all neutrosophic refined sets on $\mathcal{U}$, and let $\mathcal{SVNS}(\mathfrak{U})$ denote the class of all SVNSs on $\mathfrak{U}$. Suppose that

$$f_q : \mathcal{NRS}(\mathfrak{U}) \to \mathcal{SVNS}(\mathfrak{U})$$

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is a mapping defined on $\mathcal{NRS}(\mathcal{U})$. Then f_q is called the *neutrosophic refined power mean operator*, and it is defined as

$$f_q(\mathcal{NRS}(\sqcap)) = \left\langle \left(\frac{1}{p}\sum_{i=1}^{p}\left(\mathfrak{T}^i_{\mathcal{N}}(\sqcap)\right)^q\right)^{\frac{1}{q}}, \left(\frac{1}{p}\sum_{i=1}^{p}\left(\mathfrak{I}^i_{\mathcal{N}}(\sqcap)\right)^q\right)^{\frac{1}{q}}, \left(\frac{1}{p}\sum_{i=1}^{p}\left(\mathfrak{F}^i_{\mathcal{N}}(\sqcap)\right)^q\right)^{\frac{1}{q}} \right\rangle \tag{1}$$

where $q \geq 0$ is a real parameter and p is a positive integer. Here, $\mathfrak{T}^i_{\mathcal{N}}(\sqcap)$, $\mathfrak{I}^i_{\mathcal{N}}(\sqcap)$, and $\mathfrak{F}^i_{\mathcal{N}}(\sqcap)$ denote the i^{th} refined T-function, I-function, and F-function values of the neutrosophic refined set $\mathcal{N}$.

Lemma 1. For the operator $f_q(\mathcal{NRS}(\sqcap))$ defined above,

$$f_q(\mathcal{NRS}(\sqcap)) = \left\langle \left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{T}^i_{\mathcal{N}}(\sqcap))^q\right)^{\frac{1}{q}}, \left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{I}^i_{\mathcal{N}}(\sqcap))^q\right)^{\frac{1}{q}}, \left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{F}^i_{\mathcal{N}}(\sqcap))^q\right)^{\frac{1}{q}} \right\rangle \in \mathcal{SVNS}(\mathfrak{U}).$$

Proof. For every $\sqcap \in \mathfrak{U}$ and $i = 1, 2, \ldots, p$,

$$\mathfrak{T}^i_{\mathcal{N}}(\sqcap),\ \mathfrak{I}^i_{\mathcal{N}}(\sqcap),\ \mathfrak{F}^i_{\mathcal{N}}(\sqcap) \in [0,1].$$

Hence,

$$0 \leq \mathfrak{T}^i_{\mathcal{N}}(\sqcap) \leq 1 \quad \Rightarrow \quad 0 \leq (\mathfrak{T}^i_{\mathcal{N}}(\sqcap))^q \leq 1 \qquad (q \geq 0).$$

Thus,

$$0 \leq \sum_{i=1}^{p}(\mathfrak{T}^i_{\mathcal{N}}(\sqcap))^q \leq p \quad \Rightarrow \quad 0 \leq \frac{1}{p}\sum_{i=1}^{p}(\mathfrak{T}^i_{\mathcal{N}}(\sqcap))^q \leq 1.$$

Therefore,

$$0 \leq \left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{T}^i_{\mathcal{N}}(\sqcap))^q\right)^{\frac{1}{q}} \leq 1.$$

Similarly,

$$0 \leq \left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{I}^i_{\mathcal{N}}(\sqcap))^q\right)^{\frac{1}{q}} \leq 1, \qquad 0 \leq \left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{F}^i_{\mathcal{N}}(\sqcap))^q\right)^{\frac{1}{q}} \leq 1.$$

Hence,

$$\left\langle \left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{T}^i_{\mathcal{N}}(\sqcap))^q\right)^{\frac{1}{q}}, \left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{I}^i_{\mathcal{N}}(\sqcap))^q\right)^{\frac{1}{q}}, \left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{F}^i_{\mathcal{N}}(\sqcap))^q\right)^{\frac{1}{q}} \right\rangle \in \mathcal{SVNS}(\mathfrak{U}),$$

which completes the proof.

Special Cases.

Case 1.

For $q = 1$, Eq. (1) simplifies to

$$f_1(\mathcal{NRS}(\sqcap)) = \left\{ \frac{1}{p}\sum_{i=1}^{p}\mathfrak{T}^i_{\mathcal{N}}(\sqcap),\ \frac{1}{p}\sum_{i=1}^{p}\mathfrak{I}^i_{\mathcal{N}}(\sqcap),\ \frac{1}{p}\sum_{i=1}^{p}\mathfrak{F}^i_{\mathcal{N}}(\sqcap) \right\}, \qquad \forall \sqcap \in \mathfrak{U}. \tag{2}$$

Expression (2) is referred to as the *arithmetic mean transformation function (AMTF)*.

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Case 2.
For $q = 0$, Eq. (1) reduces to

$$\begin{aligned} f_0(\mathcal{NRS}(\sqcap)) &= \left\{ \sqrt[p]{\mathfrak{T}^1_{\mathcal{N}}(\sqcap)\cdots\mathcal{T}^p_{\mathcal{N}}(\sqcap)},\ \sqrt[p]{\mathfrak{I}^1_{\mathcal{N}}(\sqcap)\cdots\mathfrak{I}^p_{\mathcal{N}}(\sqcap)},\ \sqrt[p]{\mathfrak{F}^1_{\mathcal{N}}(\sqcap)\cdots\mathfrak{F}^p_{\mathcal{N}}(\sqcap)} \right\} \\ &= \left\{ \left(\prod_{i=1}^{p}\mathfrak{T}^i_{\mathcal{N}}(\sqcap)\right)^{1/p},\ \left(\prod_{i=1}^{p}\mathfrak{I}^i_{\mathcal{N}}(\sqcap)\right)^{1/p},\ \left(\prod_{i=1}^{p}\mathfrak{F}^i_{\mathcal{N}}(\sqcap)\right)^{1/p} \right\}, \qquad \forall \sqcap \in \mathfrak{U}. \end{aligned} \tag{3}$$

Equation (3) is known as the *geometric mean transformation function (GMTF)*.

Proof. To analyze the behavior of Eq. (1), we rewrite it in exponential form:

$$f_q(\mathcal{NRS}(\sqcap)) = \left\langle \left(\tfrac{1}{p}\sum_{i=1}^{p}(\mathfrak{T}^i_{\mathcal{N}}(\sqcap))^q\right)^{1/q},\ \left(\tfrac{1}{p}\sum_{i=1}^{p}(\mathfrak{I}^i_{\mathcal{N}}(\sqcap))^q\right)^{1/q},\ \left(\tfrac{1}{p}\sum_{i=1}^{p}(\mathfrak{F}^i_{\mathcal{N}}(\sqcap))^q\right)^{1/q} \right\rangle.$$

By expressing each component logarithmically, we obtain

$$\begin{aligned} f_q(\mathcal{NRS}(\sqcap)) = \Bigg\langle &\exp\left(\frac{1}{q}\ln\left[\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{T}^i_{\mathcal{N}}(\sqcap))^q\right]\right),\ \exp\left(\frac{1}{q}\ln\left[\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{I}^i_{\mathcal{N}}(\sqcap))^q\right]\right), \\ &\exp\left(\frac{1}{q}\ln\left[\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{F}^i_{\mathcal{N}}(\sqcap))^q\right]\right)\Bigg\rangle. \end{aligned}$$

Taking the limit $q \to 0$, L'Hospital's rule yields

$$\left\langle \sum_{i=1}^{p}\tfrac{1}{p}\ln\mathfrak{T}^i_{\mathcal{N}}(\sqcap),\ \sum_{i=1}^{p}\tfrac{1}{p}\ln\mathfrak{I}^i_{\mathcal{N}}(\sqcap),\ \sum_{i=1}^{p}\tfrac{1}{p}\ln\mathfrak{F}^i_{\mathcal{N}}(\sqcap)\right\rangle.$$

Finally, continuity of the exponential function provides

$$\lim_{q\to 0} f_q(\mathcal{NRS}(\sqcap)) = \left\langle \sqrt[p]{\prod_{i=1}^{p}\mathfrak{T}^i_{\mathcal{N}}(\sqcap)},\ \sqrt[p]{\prod_{i=1}^{p}\mathfrak{I}^i_{\mathcal{N}}(\sqcap)},\ \sqrt[p]{\prod_{i=1}^{p}\mathfrak{F}^i_{\mathcal{N}}(\sqcap)}\right\rangle = f_0(\mathcal{NRS}(\sqcap)).$$

□

Case 3.
For $q = 2, 3, \ldots$, the expression in Eq. (1) generates a sequence of higher-order refined mean operators. Specifically, for $q = 2$ we obtain the *quadratic mean transformation function*, for $q = 3$ the *cubic mean transformation function*, and similarly for all larger values of q.

Case 4. *The limit as* $q \to +\infty$

$$\begin{aligned} \lim_{q\to+\infty} f_q(\mathcal{NRS}(\sqcap)) = \Bigg\langle &\lim_{q\to+\infty}\left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{T}^i_{\mathcal{N}}(\sqcap))^q\right)^{\frac{1}{q}}, \\ &\lim_{q\to+\infty}\left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{I}^i_{\mathcal{N}}(\sqcap))^q\right)^{\frac{1}{q}}, \\ &\lim_{q\to+\infty}\left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{F}^i_{\mathcal{N}}(\sqcap))^q\right)^{\frac{1}{q}}\Bigg\rangle. \end{aligned}$$

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Without affecting generality, suppose that the refined membership values are sorted in non-increasing sequence.

$$\mathfrak{T}^1_{\mathcal{N}}(\sqcap) \geq \mathfrak{T}^2_{\mathcal{N}}(\sqcap) \geq \cdots \geq \mathfrak{T}^p_{\mathcal{N}}(\sqcap),$$
$$\mathfrak{I}^1_{\mathcal{N}}(\sqcap) \geq \mathfrak{I}^2_{\mathcal{N}}(\sqcap) \geq \cdots \geq \mathfrak{I}^p_{\mathcal{N}}(\sqcap),$$
$$\mathfrak{F}^1_{\mathcal{N}}(\sqcap) \geq \mathfrak{F}^2_{\mathcal{N}}(\sqcap) \geq \cdots \geq \mathfrak{F}^p_{\mathcal{N}}(\sqcap).$$

Then,

$$\lim_{q\to+\infty} f_q(\mathcal{NRS}(\sqcap)) = \left\langle \mathfrak{T}^1_{\mathcal{N}}(\sqcap)\left(\lim_{q\to+\infty}\left(\frac{1}{p}\sum_{i=1}^{p}\left(\frac{\mathfrak{T}^i_{\mathcal{N}}(\sqcap)}{\mathfrak{T}^1_{\mathcal{N}}(\sqcap)}\right)^q\right)\right)^{\frac{1}{q}}, \right.$$
$$\mathfrak{I}^1_{\mathcal{N}}(\sqcap)\left(\lim_{q\to+\infty}\left(\frac{1}{p}\sum_{i=1}^{p}\left(\frac{\mathfrak{I}^i_{\mathcal{N}}(\sqcap)}{\mathfrak{I}^1_{\mathcal{N}}(\sqcap)}\right)^q\right)\right)^{\frac{1}{q}},$$
$$\left.\mathfrak{F}^1_{\mathcal{N}}(\sqcap)\left(\lim_{q\to+\infty}\left(\frac{1}{p}\sum_{i=1}^{p}\left(\frac{\mathfrak{F}^i_{\mathcal{N}}(\sqcap)}{\mathfrak{F}^1_{\mathcal{N}}(\sqcap)}\right)^q\right)\right)^{\frac{1}{q}}\right\rangle.$$

Because all normalized terms lie within $[0,1]$ and the largest element dominates as $q \to +\infty$, we obtain

$$\lim_{q\to+\infty} f_q(\mathcal{NRS}(\sqcap)) = \left\langle \mathcal{T}^1_{\mathcal{N}}(\sqcap),\ \mathcal{I}^1_{\mathcal{N}}(\sqcap),\ \mathcal{F}^1_{\mathcal{N}}(\sqcap)\right\rangle.$$

Equivalently,

$$\lim_{q\to+\infty} f_q(\mathcal{NRS}(\sqcap)) = \left\langle \max\{\mathfrak{T}^1_{\mathcal{N}}(\sqcap),\ldots,\mathfrak{T}^p_{\mathcal{N}}(\sqcap)\},\ \max\{\mathfrak{I}^1_{\mathcal{N}}(\sqcap),\ldots,\mathfrak{I}^p_{\mathcal{N}}(\sqcap)\},\right.$$
$$\left.\max\{\mathfrak{F}^1_{\mathcal{N}}(\sqcap),\ldots,\mathfrak{F}^p_{\mathcal{N}}(\sqcap)\}\right\rangle. \tag{4}$$

3. Properties of Neutrosophic Refined Power Mean Operator

Property 1: Idempotency

If all refined T-function values $\{\mathfrak{T}^i_{\mathcal{N}}(\sqcap)\}$, all refined I-function values $\{\mathfrak{I}^i_{\mathcal{N}}(\sqcap)\}$, and all refined F-function values $\{\mathfrak{F}^i_{\mathcal{N}}(\sqcap)\}$ are constant, then the operator f_q returns that same constant triple.

$$f_q(\mathcal{NRS}(\sqcap)) = \left\langle \left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{T}^i_{\mathcal{N}}(\sqcap))^q\right)^{\frac{1}{q}}, \left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{I}^i_{\mathcal{N}}(\sqcap))^q\right)^{\frac{1}{q}}, \left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{F}^i_{\mathcal{N}}(\sqcap))^q\right)^{\frac{1}{q}}\right\rangle.$$

Proof. Assume that the refined membership sequences consist of identical values, i.e.,

$$\mathfrak{T}^1_{\mathcal{N}}(\sqcap) = \mathfrak{T}^2_{\mathcal{N}}(\sqcap) = \cdots = \mathfrak{T}^p_{\mathcal{N}}(\sqcap) = \mathfrak{T},$$
$$\mathfrak{I}^1_{\mathcal{N}}(\sqcap) = \mathfrak{I}^2_{\mathcal{N}}(\sqcap) = \cdots = \mathfrak{I}^p_{\mathcal{N}}(\sqcap) = \mathfrak{I},$$
$$\mathfrak{F}^1_{\mathcal{N}}(\sqcap) = \mathfrak{F}^2_{\mathcal{N}}(\sqcap) = \cdots = \mathfrak{F}^p_{\mathcal{N}}(\sqcap) = \mathfrak{F}.$$

Then each summation in the operator becomes

$$\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{T}^i_{\mathcal{N}}(\sqcap))^q = \frac{1}{p}\cdot p\cdot\mathfrak{T}^q = \mathfrak{T}^q.$$

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and similarly for the indeterminacy and falsity components. Thus,

$$f_q(\mathcal{NRS}(\sqcap)) = \left\langle (\mathfrak{T}^q)^{\frac{1}{q}},\ (\mathfrak{I}^q)^{\frac{1}{q}},\ (\mathfrak{F}^q)^{\frac{1}{q}} \right\rangle = \langle \mathfrak{T},\ \mathfrak{I},\ \mathfrak{F} \rangle.$$

Hence, the operator f_q is idempotent, completing the proof.

Property 2: Monotonicity

Let

$$\mathcal{N}_1 = \{\sqcap,\ \langle \mathfrak{T}^i_{\mathcal{N}_1}(\sqcap), \mathfrak{I}^i_{\mathcal{N}_1}(\sqcap), \mathfrak{F}^i_{\mathcal{N}_1}(\sqcap)\rangle :\ \sqcap \in \mathcal{U},\ i = 1, 2, \ldots, p\}$$

and

$$\mathcal{N}_2 = \{\sqcap,\ \langle \mathfrak{T}^i_{\mathcal{N}_2}(\sqcap), \mathfrak{I}^i_{\mathcal{N}_2}(\sqcap), \mathfrak{F}^i_{\mathcal{N}_2}(\sqcap)\rangle :\ \sqcap \in \mathcal{U},\ i = 1, 2, \ldots, p\}$$

be two NRSs defined on $\mathcal{U}$.

Assume that $\mathcal{N}_1 \leq \mathcal{N}_2$ componentwise, i.e.,

$$\mathfrak{T}^i_{\mathcal{N}_1}(\sqcap) \leq \mathfrak{T}^i_{\mathcal{N}_2}(\sqcap), \qquad \mathfrak{I}^i_{\mathcal{N}_1}(\sqcap) \geq \mathfrak{I}^i_{\mathcal{N}_2}(\sqcap), \qquad \mathfrak{F}^i_{\mathcal{N}_1}(\sqcap) \geq \mathfrak{F}^i_{\mathcal{N}_2}(\sqcap),$$

for every $i = 1, 2, \ldots, p$ and $\sqcap \in \mathcal{U}$. Then,

$$f_q(\mathcal{N}_1) \leq f_q(\mathcal{N}_2).$$

$$\begin{aligned} f_q(\mathcal{NRS}(\mathcal{N}_1(\sqcap))) &= \left\langle \left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{T}^i_{\mathcal{N}_1}(\sqcap))^q\right)^{\frac{1}{q}}, \left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{I}^i_{\mathcal{N}_1}(\sqcap))^q\right)^{\frac{1}{q}}, \left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{F}^i_{\mathcal{N}_1}(\sqcap))^q\right)^{\frac{1}{q}} \right\rangle \\ &\leq \left\langle \left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{T}^i_{\mathcal{N}_2}(\sqcap))^q\right)^{\frac{1}{q}}, \left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{I}^i_{\mathcal{N}_2}(\sqcap))^q\right)^{\frac{1}{q}}, \left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{F}^i_{\mathcal{N}_2}(\sqcap))^q\right)^{\frac{1}{q}} \right\rangle \\ &= f_q(\mathcal{NRS}(\mathcal{N}_2(\sqcap))). \end{aligned} \tag{5}$$

Proof. Since

$$\mathfrak{T}^i_{\mathcal{N}_1}(\sqcap) \leq \mathfrak{T}^i_{\mathcal{N}_2}(\sqcap),$$

raising both sides to the power q gives

$$(\mathfrak{T}^i_{\mathcal{N}_1}(\sqcap))^q \leq (\mathfrak{T}^i_{\mathcal{N}_2}(\sqcap))^q.$$

Summing over i yields

$$\sum_{i=1}^{p}(\mathfrak{T}^i_{\mathcal{N}_1}(\sqcap))^q \leq \sum_{i=1}^{p}(\mathfrak{T}^i_{\mathcal{N}_2}(\sqcap))^q,$$

and hence

$$\left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{T}^i_{\mathcal{N}_1}(\sqcap))^q\right)^{\frac{1}{q}} \leq \left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{T}^i_{\mathcal{N}_2}(\sqcap))^q\right)^{\frac{1}{q}}. \tag{6}$$

Similarly, from $\mathfrak{I}^i_{\mathcal{N}_1}(\sqcap) \geq \mathfrak{I}^i_{\mathcal{N}_2}(\sqcap)$ and $\mathfrak{F}^i_{\mathcal{N}_1}(\sqcap) \geq \mathfrak{F}^i_{\mathcal{N}_2}(\sqcap)$, we obtain

$$\left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{I}^i_{\mathcal{N}_1}(\sqcap))^q\right)^{\frac{1}{q}} \geq \left(\frac{1}{p}\sum_{i=1}^{p}(\mathfrak{I}^i_{\mathcal{N}_2}(\sqcap))^q\right)^{\frac{1}{q}}, \tag{7}$$

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$$\left(\frac{1}{p}\sum_{i=1}^{p}\left(\mathfrak{F}^{i}_{\mathcal{N}_1}(\sqcap)\right)^{q}\right)^{\frac{1}{q}} \geq \left(\frac{1}{p}\sum_{i=1}^{p}\left(\mathfrak{F}^{i}_{\mathcal{N}_2}(\sqcap)\right)^{q}\right)^{\frac{1}{q}}. \tag{8}$$

Combining (6)–(8), we conclude that

$$f_q(\mathcal{NRS}(\mathcal{N}_1(\sqcap))) \leq f_q(\mathcal{NRS}(\mathcal{N}_2(\sqcap))).$$

which proves the monotonicity of the operator.

Property 3: Boundedness

Let $\mathcal{N} = \mathcal{NRS}(\mathfrak{U})$ be any family of neutrosophic refined sets defined on the universe $\mathfrak{U}$. For this collection, define the upper and lower neutrosophic bounds as

$$\mathcal{N}^{+} = \left\langle \max_i\{\mathfrak{T}^{i}_{\mathcal{N}}\},\ \min_i\{\mathfrak{I}^{i}_{\mathcal{N}}\},\ \min_i\{\mathfrak{F}^{i}_{\mathcal{N}}\}\right\rangle, \qquad \mathcal{N}^{-} = \left\langle \min_i\{\mathfrak{T}^{i}_{\mathcal{N}}\},\ \max_i\{\mathfrak{I}^{i}_{\mathcal{N}}\},\ \max_i\{\mathfrak{F}^{i}_{\mathcal{N}}\}\right\rangle.$$

Then the following bound holds:

$$\mathcal{N}^{-} \ \leq\ f_q(\mathcal{NRS}(\mathcal{N}(\sqcap))) \ \leq\ \mathcal{N}^{+}.$$

Proof.

From Properties 1 and 2, each component produced by the operator f_q is bounded below by the corresponding component of $\mathcal{N}^{-}$ and bounded above by the corresponding component of $\mathcal{N}^{+}$. Thus,

$$f_q(\mathcal{NRS}(\mathcal{N}(\sqcap))) \geq \mathcal{N}^{-}, \qquad f_q(\mathcal{NRS}(\mathcal{N}(\sqcap))) \leq \mathcal{N}^{+},$$

which directly yields

$$\mathcal{N}^{-} \ \leq\ f_q(\mathcal{NRS}(\mathcal{N}(\sqcap))) \ \leq\ \mathcal{N}^{+}.$$

4. CEM for NRSs

Here, two distinct formulations are proposed to evaluate the CEM of NRSs. These formulations are developed by extending the concept of the CEM defined for SVNs, as introduced in [11]. For completeness, we first revisit the notion of cross-entropy within the framework of SVNs.

Definition 4: Single-valued neutrosophic CEM [11].

Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be two SVNSs defined over the universe

$$\mathfrak{U} = \{\sqcap_1, \sqcap_2, \sqcap_3, \ldots, \sqcap_n\}.$$

The CEM between $\mathcal{A}_1$ and $\mathcal{A}_2$ is represented by

$$CE_{\mathcal{SVNS}}(\mathcal{A}_1, \mathcal{A}_2),$$

and is mathematically expressed as

$$\begin{aligned} CE_{\mathcal{SVNS}}(\mathcal{A}_1, \mathcal{A}_2) = \frac{1}{2n}\sum_{i=1}^{p}\Bigg[& \frac{\left|\, 2\mathfrak{T}_{\mathcal{A}_1}(\sqcap_i) - \mathfrak{T}_{\mathcal{A}_2}(\sqcap_i)\,\right| + \left|\, 2\left(1-\mathfrak{T}_{\mathcal{A}_1}(\sqcap_i)\right) - \left(1-\mathfrak{T}_{\mathcal{A}_2}(\sqcap_i)\right)\,\right|}{\sqrt{1+\left|\mathfrak{T}_{\mathcal{A}_1}(\sqcap_i)\right|^2} + \sqrt{1+\left|\mathfrak{T}_{\mathcal{A}_2}(\sqcap_i)\right|^2}} \\ & + \frac{\left|\, 2\mathfrak{I}_{\mathcal{A}_1}(\sqcap_i) - \mathfrak{I}_{\mathcal{A}_2}(\sqcap_i)\,\right| + \left|\, 2\left(1-\mathfrak{I}_{\mathcal{A}_1}(\sqcap_i)\right) - \left(1-\mathfrak{I}_{\mathcal{A}_2}(\sqcap_i)\right)\,\right|}{\sqrt{1+\left|\mathfrak{I}_{\mathcal{A}_1}(\sqcap_i)\right|^2} + \sqrt{1+\left|\mathfrak{I}_{\mathcal{A}_2}(\sqcap_i)\right|^2}} \\ & + \frac{\left|\, 2\mathfrak{F}_{\mathcal{A}_1}(\sqcap_i) - \mathfrak{F}_{\mathcal{A}_2}(\sqcap_i)\,\right| + \left|\, 2\left(1-\mathfrak{F}_{\mathcal{A}_1}(\sqcap_i)\right) - \left(1-\mathfrak{F}_{\mathcal{A}_2}(\sqcap_i)\right)\,\right|}{\sqrt{1+\left|\mathfrak{F}_{\mathcal{A}_1}(\sqcap_i)\right|^2} + \sqrt{1+\left|\mathfrak{F}_{\mathcal{A}_2}(\sqcap_i)\right|^2}} \Bigg] \end{aligned} \tag{9}$$

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Theorem 1: *Single-valued neutrosophic CEM* $CE_{\mathcal{SVNS}}(\mathcal{A}_1, \mathcal{A}_2)$ for any two SVNSs $\mathcal{A}_1$ and $\mathcal{A}_2$ meets the following conditions:

i. $CE_{\mathcal{SVNS}}(\mathcal{A}_1, \mathcal{A}_2) \geq 0, \quad \forall \sqcap_i \in \mathcal{U}.$

ii. $CE_{\mathcal{SVNS}}(\mathcal{A}_1, \mathcal{A}_2) = 0$ if and only if $\mathfrak{T}_{\mathcal{A}_1}(\sqcap_i) = \mathfrak{T}_{\mathcal{A}_2}(\sqcap_i)$, $\mathfrak{I}_{\mathcal{A}_1}(\sqcap_i) = \mathcal{I}_{\mathcal{A}_2}(\sqcap_i)$, $\mathfrak{F}_{\mathcal{A}_1}(\sqcap_i) = \mathcal{F}_{\mathcal{A}_2}(\sqcap_i)$, $\forall \sqcap_i \in \mathfrak{U}$.

iii. $CE_{SVNS}(\mathcal{A}_1, \mathcal{A}_2) = CE_{SVNS}(\mathcal{A}_1^c, \mathcal{A}_2^c), \quad \forall \sqcap_i \in \mathfrak{U}.$

iv. $CE_{SVNS}(\mathcal{A}_1, \mathcal{A}_2) = CE_{SVNS}(\mathcal{A}_2, \mathcal{A}_1), \quad \forall \sqcap_i \in \mathfrak{U}.$

Definition 5: Neutrosophic Refined CEM

Let $\mathcal{N}_1$ and $\mathcal{N}_2$ be any two NRSs in $U = \{u_1, u_2, u_3, \ldots, u_n\}$. Then, the neutrosophic refined cross-entropy between $\mathcal{N}_1$ and $\mathcal{N}_2$ is defined as

$$
\begin{aligned}
&CE_{NRS}(\mathcal{N}_1, \mathcal{N}_2) = \\
&\frac{1}{2p}\sum_{i=1}^{n}\Bigg\{\Bigg[\frac{2\left|\mathfrak{T}^1_{N_1}(u_i) - \mathfrak{T}^1_{N_2}(u_i)\right|}{\sqrt{1+\left|\mathfrak{T}^1_{N_1}(u_i)\right|^2}+\sqrt{1+\left|\mathfrak{T}^1_{N_2}(u_i)\right|^2}} + \frac{2\left|(1-\mathfrak{T}^1_{N_1}(u_i)) - (1-\mathfrak{T}^1_{N_2}(u_i))\right|}{\sqrt{1+\left|(1-\mathfrak{T}^1_{N_1}(u_i))\right|^2}+\sqrt{1+\left|(1-\mathfrak{T}^1_{N_2}(u_i))\right|^2}}\Bigg] \\
&+\Bigg[\frac{2\left|\mathfrak{T}^2_{N_1}(u_i) - \mathfrak{T}^2_{N_2}(u_i)\right|}{\sqrt{1+\left|\mathfrak{T}^2_{N_1}(u_i)\right|^2}+\sqrt{1+\left|\mathfrak{T}^2_{N_2}(u_i)\right|^2}} + \frac{2\left|(1-\mathfrak{T}^2_{N_1}(u_i)) - (1-\mathfrak{T}^2_{N_2}(u_i))\right|}{\sqrt{1+\left|(1-\mathfrak{T}^2_{N_1}(u_i))\right|^2}+\sqrt{1+\left|(1-\mathfrak{T}^2_{N_2}(u_i))\right|^2}}\Bigg] + \ldots \\
&+\Bigg[\frac{2\left|\mathfrak{T}^p_{N_1}(u_i) - \mathfrak{T}^p_{N_2}(u_i)\right|}{\sqrt{1+\left|\mathfrak{T}^p_{N_1}(u_i)\right|^2}+\sqrt{1+\left|\mathfrak{T}^p_{N_2}(u_i)\right|^2}} + \frac{2\left|(1-\mathfrak{T}^p_{N_1}(u_i)) - (1-\mathfrak{T}^p_{N_2}(u_i))\right|}{\sqrt{1+\left|(1-\mathfrak{T}^p_{N_1}(u_i))\right|^2}+\sqrt{1+\left|(1-\mathfrak{T}^p_{N_2}(u_i))\right|^2}}\Bigg] \\
&+\Bigg[\frac{2\left|\mathfrak{I}^1_{N_1}(u_i) - \mathfrak{I}^1_{N_2}(u_i)\right|}{\sqrt{1+\left|\mathfrak{I}^1_{N_1}(u_i)\right|^2}+\sqrt{1+\left|\mathfrak{I}^1_{N_2}(u_i)\right|^2}} + \frac{2\left|(1-\mathfrak{I}^1_{N_1}(u_i)) - (1-\mathfrak{I}^1_{N_2}(u_i))\right|}{\sqrt{1+\left|(1-\mathfrak{I}^1_{N_1}(u_i))\right|^2}+\sqrt{1+\left|(1-\mathfrak{I}^1_{N_2}(u_i))\right|^2}}\Bigg] + \ldots \\
&+\Bigg[\frac{2\left|\mathfrak{F}^1_{N_1}(u_i) - \mathfrak{F}^1_{N_2}(u_i)\right|}{\sqrt{1+\left|\mathfrak{F}^1_{N_1}(u_i)\right|^2}+\sqrt{1+\left|\mathfrak{F}^1_{N_2}(u_i)\right|^2}} + \frac{2\left|(1-\mathfrak{F}^1_{N_1}(u_i)) - (1-\mathfrak{F}^1_{N_2}(u_i))\right|}{\sqrt{1+\left|(1-\mathfrak{F}^1_{N_1}(u_i))\right|^2}+\sqrt{1+\left|(1-\mathfrak{F}^1_{N_2}(u_i))\right|^2}}\Bigg] + \ldots\Bigg\}.
\end{aligned}
\tag{10}
$$

Theorem 2: *Neutrosophic refined CEM* $CE_{NRS}(N_1, N_2)$ *for any two NRSs* N_1, N_2 *satisfies the following properties:*

i. $CE_{NRS}(N_1, N_2) \geq 0, \ \forall u_i \in \mathfrak{U}.$

ii. $CE_{NRS}(N_1, N_2) = 0$ *if and only if* $\mathfrak{T}^i_{N_1}(u_i) = \mathfrak{T}^i_{N_2}(u_i)$, $\mathfrak{I}^i_{N_1}(u_i) = \mathfrak{I}^i_{N_2}(u_i)$, $\mathfrak{F}^i_{N_1}(u_i) = \mathfrak{F}^i_{N_2}(u_i)$, $\forall u_i \in \mathfrak{U}.$

iii. $CE_{NRS}(N_1, N_2) = CE_{NRS}(N_1^c, N_2^c), \ \forall u_i \in \mathfrak{U}.$

iv. $CE_{NRS}(N_1, N_2) = CE_{NRS}(N_2, N_1), \ \forall u_i \in \mathfrak{U}.$

Definition 6: Neutrosophic refined weighted CEM

Let $\mathcal{N}_1$ and $\mathcal{N}_2$ be two NRSs defined throughout the universe $\mathfrak{U} = \{\sqcap_1, \sqcap_2, \sqcap_3, \ldots, \sqcap_n\}$. Assume that each element $\sqcap_i \in \mathfrak{U}$ is assigned a weight w_i, forming the weight vector $\mathcal{W} = \{w_1, w_2, \ldots, w_n\}$, where $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$. Under these assumptions, the weighted cross-entropy measure for $\mathcal{N}_1$ and $\mathcal{N}_2$ in the neutrosophic refined framework is defined as follows:

$$CE^{\mathcal{W}}_{NRS}(\mathcal{N}_1, \mathcal{N}_2) = \frac{1}{2p}\{\sum_{i=1}^{n} w_i[(\frac{2|\mathfrak{T}^1_{\mathcal{N}_1}(\sqcap_i) - \mathfrak{T}^1_{\mathcal{N}_2}(\sqcap_i)|}{\sqrt{1+|\mathfrak{T}^1_{\mathcal{N}_1}(\sqcap_i)|^2}+\sqrt{1+|\mathfrak{T}^1_{\mathcal{N}_2}(\sqcap_i)|^2}} + \frac{2|(1-\mathfrak{T}^1_{\mathcal{N}_1}(\sqcap_i)) - (1-\mathfrak{T}^1_{\mathcal{N}_2}(\sqcap_i))|}{\sqrt{1+|(1-\mathfrak{T}^1_{\mathcal{N}_1}(\sqcap_i))|^2}+\sqrt{1+|(1-\mathfrak{T}^1_{\mathcal{N}_2}(\sqcap_i))|^2}})]+$$

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$$\left[\frac{2\left|\mathfrak{T}^2_{\mathcal{N}_1}(\sqcap_i)-\mathfrak{T}^2_{\mathcal{N}_2}(\sqcap_i)\right|}{\sqrt{1+\left|\mathfrak{T}^2_{\mathcal{N}_1}(\sqcap_i)\right|^2}+\sqrt{1+\left|\mathfrak{T}^2_{\mathcal{N}_2}(\sqcap_i)\right|^2}}+\frac{2\left|(1-\mathfrak{T}^2_{\mathcal{N}_1}(\sqcap_i))-(1-\mathfrak{T}^2_{\mathcal{N}_2}(\sqcap_i))\right|}{\sqrt{1+\left|(1-\mathfrak{T}^2_{\mathcal{N}_1}(\sqcap_i))\right|^2}+\sqrt{1+\left|(1-\mathfrak{T}^2_{\mathcal{N}_2}(\sqcap_i))\right|^2}}+\ldots\right]$$

$$+\left[\frac{2\left|\mathfrak{T}^{\mathfrak{p}}_{\mathcal{N}_1}(\sqcap_i)-\mathfrak{T}^{\mathfrak{p}}_{\mathcal{N}_2}(\sqcap_i)\right|}{\sqrt{1+\left|\mathfrak{T}^{\mathfrak{p}}_{\mathcal{N}_1}(\sqcap_i)\right|^2}+\sqrt{1+\left|\mathfrak{T}^{\mathfrak{p}}_{\mathcal{N}_2}(\sqcap_i)\right|^2}}+\frac{2\left|(1-\mathfrak{T}^{\mathfrak{p}}_{\mathcal{N}_1}(\sqcap_i))-(1-\mathfrak{T}^{\mathfrak{p}}_{\mathcal{N}_2}(\sqcap_i))\right|}{\sqrt{1+\left|(1-\mathfrak{T}^{\mathfrak{p}}_{\mathcal{N}_1}(\sqcap_i))\right|^2}+\sqrt{1+\left|(1-\mathfrak{T}^{\mathfrak{p}}_{\mathcal{N}_2}(\sqcap_i))\right|^2}}\right]$$

$$+\left[\frac{2\left|\mathfrak{I}^1_{\mathcal{N}_1}(\sqcap_i)-\mathfrak{I}^1_{\mathcal{N}_2}(\sqcap_i)\right|}{\sqrt{1+\left|\mathfrak{I}^1_{\mathcal{N}_1}(\sqcap_i)\right|^2}+\sqrt{1+\left|\mathfrak{I}^1_{\mathcal{N}_2}(\sqcap_i)\right|^2}}+\frac{2\left|(1-\mathfrak{I}^1_{\mathcal{N}_1}(\sqcap_i))-(1-\mathfrak{I}^1_{\mathcal{N}_2}(\sqcap_i))\right|}{\sqrt{1+\left|(1-\mathfrak{I}^1_{\mathcal{N}_1}(\sqcap_i))\right|^2}+\sqrt{1+\left|(1-\mathfrak{I}^1_{\mathcal{N}_2}(\sqcap_i))\right|^2}}\right] \tag{11}$$

$$+\left[\frac{2\left|\mathfrak{I}^2_{\mathcal{N}_1}(\sqcap_i)-\mathfrak{I}^2_{\mathcal{N}_2}(\sqcap_i)\right|}{\sqrt{1+\left|\mathfrak{I}^2_{\mathcal{N}_1}(\sqcap_i)\right|^2}+\sqrt{1+\left|\mathfrak{I}^2_{\mathcal{N}_2}(\sqcap_i)\right|^2}}+\frac{2\left|(1-\mathfrak{I}^2_{\mathcal{N}_1}(\sqcap_i))-(1-\mathfrak{I}^2_{\mathcal{N}_2}(\sqcap_i))\right|}{\sqrt{1+\left|(1-\mathfrak{I}^2_{\mathcal{N}_1}(\sqcap_i))\right|^2}+\sqrt{1+\left|(1-\mathfrak{I}^2_{\mathcal{N}_2}(\sqcap_i))\right|^2}}+\ldots\right]$$

$$+\left[\frac{2\left|\mathfrak{I}^{\mathfrak{p}}_{\mathcal{N}_1}(\sqcap_i)-\mathfrak{I}^{\mathfrak{p}}_{\mathcal{N}_2}(\sqcap_i)\right|}{\sqrt{1+\left|\mathfrak{I}^{\mathfrak{p}}_{\mathcal{N}_1}(\sqcap_i)\right|^2}+\sqrt{1+\left|\mathfrak{I}^{\mathfrak{p}}_{\mathcal{N}_2}(\sqcap_i)\right|^2}}+\frac{2\left|(1-\mathfrak{I}^{\mathfrak{p}}_{\mathcal{N}_1}(\sqcap_i))-(1-\mathfrak{I}^{\mathfrak{p}}_{\mathcal{N}_2}(\sqcap_i))\right|}{\sqrt{1+\left|(1-\mathfrak{I}^{\mathfrak{p}}_{\mathcal{N}_1}(\sqcap_i))\right|^2}+\sqrt{1+\left|(1-\mathfrak{I}^{\mathfrak{p}}_{\mathcal{N}_2}(\sqcap_i))\right|^2}}\right]$$

Theorem 3: The neutrosophic refined CEM $CE^{\mathcal{W}}_{\mathcal{NRS}}(\mathcal{N}_1,\mathcal{N}_2)$ for any two *NRSs* $\mathcal{N}_1,\mathcal{N}_2$ satisfies the following properties:

i. $CE^{\mathcal{W}}_{\mathcal{NRS}}(\mathcal{N}_1,\mathcal{N}_2)\geq 0,\ \forall\sqcap_i\in\mathfrak{U}.$

ii. $CE^{\mathcal{W}}_{\mathcal{NRS}}(\mathcal{N}_1,\mathcal{N}_2)=0$ if and only if

$$\mathfrak{T}^i_{\mathcal{N}_1}(\sqcap_i)=\mathfrak{T}^i_{\mathcal{N}_2}(\sqcap_i),\quad \mathfrak{I}^i_{\mathcal{N}_1}(\sqcap_i)=\mathfrak{I}^i_{\mathcal{N}_2}(\sqcap_i),\quad \mathfrak{F}^i_{\mathcal{N}_1}(\sqcap_i)=\mathfrak{F}^i_{\mathcal{N}_2}(\sqcap_i),\quad \forall\sqcap_i\in\mathfrak{U}.$$

iii. $CE^{\mathcal{W}}_{\mathcal{NRS}}(\mathcal{N}_1,\mathcal{N}_2)=CE^{\mathcal{W}}_{\mathcal{NRS}}(\mathcal{N}^c_1,\mathcal{N}^c_2),\ \forall\sqcap_i\in\mathfrak{U}.$

iv. $CE^{\mathcal{W}}_{\mathcal{NRS}}(\mathcal{N}_1,\mathcal{N}_2)=CE^{\mathcal{W}}_{\mathcal{NRS}}(\mathcal{N}_2,\mathcal{N}_1),\ \forall\sqcap_i\in\mathfrak{U}.$

5. Cross Entropy Based MADM Strategy in NRS Environment Using Power Mean Operator

In this section, a systematic procedure is presented to identify the priority ranking of all available alternatives in the NRN framework. Let $\{\mathbb{A}^{\mathbf{S}}_1,\mathbb{A}^{\mathbf{S}}_2,\ldots,\mathbb{A}^{\mathbf{S}}_m\}$ denote the collection of aspirant candidates. $\{\mathsf{C}^{\mathbf{S}}_1,\mathsf{C}^{\mathbf{S}}_2,\ldots,\mathsf{C}^{\mathbf{S}}_n\}$ represent the set of evaluation attributes and $\{\mathbf{B}^{\mathbf{s}}_1,\mathbf{B}^{\mathbf{s}}_2,\ldots,\mathbf{B}^{\mathbf{s}}_k\}$ be the set of alternatives available associated with each candidate.

The decision-maker evaluates each available alternative $\mathbb{A}^{\mathbf{S}}_i$ $(i=1,2,\ldots,m)$ with respect to each criterion that is sufficient to evaluate the alternatives are denoted by $\mathsf{C}^{\mathbf{S}}_j$ $(j=1,2,\ldots,n)$ employing *NRSs*. Moreover, the ideal performance levels of all criteria are determined by the decision-maker in the form of SVNN.

Based on the defined aggregation operators, the MADM Technique is implemented through the following steps:

Step 1: Formulation of the decision matrix (D-Matrix) relates the candidates to attributes. The relationship between each candidate $\mathbb{A}^{\mathbf{S}}_i$ $(i=1,2,\ldots,m)$ and each attribute $\mathsf{C}^{\mathbf{S}}_j$ $(j=1,2,\ldots,n)$ expressed in terms of NRS information is organized in the following matrix (denoted by $\mathcal{M}^1$):

$$\mathcal{M}^1=\text{Candidates–attributes neutrosophic refined decision-making matrix.}$$

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$$\mathcal{M}^1 = \begin{array}{c|ccc} & \mathsf{C}_1^{\mathbf{S}} & \mathsf{C}_2^{\mathbf{S}} & \cdots & \mathsf{C}_n^{\mathbf{S}} \\ \hline \mathbb{A}_1^{\mathbf{S}} & \left\langle \begin{array}{c} (\mathfrak{T}_{11}^1(\sqcap),\mathfrak{I}_{11}^1(\sqcap),\mathfrak{F}_{11}^1(\sqcap)) \\ (\mathfrak{T}_{11}^2(\sqcap),\mathfrak{I}_{11}^2(\sqcap),\mathfrak{F}_{11}^2(\sqcap)) \\ \vdots \\ (\mathfrak{T}_{11}^p(\sqcap),\mathfrak{I}_{11}^p(\sqcap),\mathfrak{F}_{11}^p(\sqcap)) \end{array} \right\rangle & \left\langle \begin{array}{c} (\mathfrak{T}_{12}^1(\sqcap),\mathfrak{I}_{12}^1(\sqcap),\mathfrak{F}_{12}^1(\sqcap)) \\ (\mathfrak{T}_{12}^2(\sqcap),\mathfrak{I}_{12}^2(\sqcap),\mathfrak{F}_{12}^2(\sqcap)) \\ \vdots \\ (\mathfrak{T}_{12}^p(\sqcap),\mathfrak{I}_{12}^p(\sqcap),\mathfrak{F}_{12}^p(\sqcap)) \end{array} \right\rangle & \cdots & \left\langle \begin{array}{c} (\mathfrak{T}_{1n}^1(\sqcap),\mathfrak{I}_{1n}^1(\sqcap),\mathfrak{F}_{1n}^1(\sqcap)) \\ (\mathfrak{T}_{1n}^2(\sqcap),\mathfrak{I}_{1n}^2(\sqcap),\mathfrak{F}_{1n}^2(\sqcap)) \\ \vdots \\ (\mathfrak{T}_{1n}^p(\sqcap),\mathfrak{I}_{1n}^p(\sqcap),\mathfrak{F}_{1n}^p(\sqcap)) \end{array} \right\rangle \\ \mathbb{A}_2^{\mathbf{S}} & \left\langle \begin{array}{c} (\mathfrak{T}_{21}^1(\sqcap),\mathfrak{I}_{21}^1(\sqcap),\mathfrak{F}_{21}^1(\sqcap)) \\ (\mathfrak{T}_{21}^2(\sqcap),\mathfrak{I}_{21}^2(\sqcap),\mathfrak{F}_{21}^2(\sqcap)) \\ \vdots \\ (\mathfrak{T}_{21}^p(\sqcap),\mathfrak{I}_{21}^p(\sqcap),\mathfrak{F}_{21}^p(\sqcap)) \end{array} \right\rangle & \left\langle \begin{array}{c} (\mathfrak{T}_{22}^1(\sqcap),\mathfrak{I}_{22}^1(\sqcap),\mathfrak{F}_{22}^1(\sqcap)) \\ (\mathfrak{T}_{22}^2(\sqcap),\mathfrak{I}_{22}^2(\sqcap),\mathfrak{F}_{22}^2(\sqcap)) \\ \vdots \\ (\mathfrak{T}_{22}^p(\sqcap),\mathfrak{I}_{22}^p(\sqcap),\mathfrak{F}_{22}^p(\sqcap)) \end{array} \right\rangle & \cdots & \left\langle \begin{array}{c} (\mathfrak{T}_{2n}^1(\sqcap),\mathfrak{I}_{2n}^1(\sqcap),\mathfrak{F}_{2n}^1(\sqcap)) \\ (\mathfrak{T}_{2n}^2(\sqcap),\mathfrak{I}_{2n}^2(\sqcap),\mathfrak{F}_{2n}^2(\sqcap)) \\ \vdots \\ (\mathfrak{T}_{2n}^p(\sqcap),\mathfrak{I}_{2n}^p(\sqcap),\mathfrak{F}_{2n}^p(\sqcap)) \end{array} \right\rangle \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbb{A}_m^{\mathbf{S}} & \left\langle \begin{array}{c} (\mathfrak{T}_{m1}^1(\sqcap),\mathfrak{I}_{m1}^1(\sqcap),\mathfrak{F}_{m1}^1(\sqcap)) \\ (\mathfrak{T}_{m1}^2(\sqcap),\mathfrak{I}_{m1}^2(\sqcap),\mathfrak{F}_{m1}^2(\sqcap)) \\ \vdots \\ (\mathfrak{T}_{m1}^p(\sqcap),\mathfrak{I}_{m1}^p(\sqcap),\mathfrak{F}_{m1}^p(\sqcap)) \end{array} \right\rangle & \left\langle \begin{array}{c} (\mathfrak{T}_{m2}^1(\sqcap),\mathfrak{I}_{m2}^1(\sqcap),\mathfrak{F}_{m2}^1(\sqcap)) \\ (\mathfrak{T}_{m2}^2(\sqcap),\mathfrak{I}_{m2}^2(\sqcap),\mathfrak{F}_{m2}^2(\sqcap)) \\ \vdots \\ (\mathfrak{T}_{m2}^p(\sqcap),\mathfrak{I}_{m2}^p(\sqcap),\mathfrak{F}_{m2}^p(\sqcap)) \end{array} \right\rangle & \cdots & \left\langle \begin{array}{c} (\mathfrak{T}_{mn}^1(\sqcap),\mathfrak{I}_{mn}^1(\sqcap),\mathfrak{F}_{mn}^1(\sqcap)) \\ (\mathfrak{T}_{mn}^2(\sqcap),\mathfrak{I}_{mn}^2(\sqcap),\mathfrak{F}_{mn}^2(\sqcap)) \\ \vdots \\ (\mathfrak{T}_{mn}^p(\sqcap),\mathfrak{I}_{mn}^p(\sqcap),\mathfrak{F}_{mn}^p(\sqcap)) \end{array} \right\rangle \end{array} \tag{12}$$

Step 2: Construction of attributes–alternative D-Matrix

The association between each attribute $\mathsf{C}_j^{\mathbf{S}}$ $(\mathsf{j} = 1, 2, \ldots, \mathsf{n})$ and each available alternative $\mathbf{B}_t^{\mathbf{s}}$ $(\mathsf{t} = 1, 2, \ldots, \mathsf{k})$ is organized in the following form, represented by the matrix $\mathcal{M}^2$.

$\mathcal{M}^2$: **Attributes versus alternatives SVN's D-Matrix**

	$\mathbf{B}_1^{\mathbf{s}}$	$\mathbf{B}_2^{\mathbf{s}}$	$\cdots$	$\mathbf{B}_k^{\mathbf{s}}$
$\mathsf{C}_1^{\mathbf{S}}$	$\langle \mathfrak{T}_{11}(\sqcap), \mathfrak{I}_{11}(\sqcap), \mathfrak{F}_{11}(\sqcap) \rangle$	$\langle \mathfrak{T}_{12}(\sqcap), \mathfrak{I}_{12}(\sqcap), \mathfrak{F}_{12}(\sqcap) \rangle$	$\cdots$	$\langle \mathfrak{T}_{1k}(\sqcap), \mathfrak{I}_{1k}(\sqcap), \mathfrak{F}_{1k}(\sqcap) \rangle$
$\mathsf{C}_2^{\mathbf{S}}$	$\langle \mathfrak{T}_{21}(\sqcap), \mathfrak{I}_{21}(\sqcap), \mathfrak{F}_{21}(\sqcap) \rangle$	$\langle \mathfrak{T}_{22}(\sqcap), \mathfrak{I}_{22}(\sqcap), \mathfrak{F}_{22}(\sqcap) \rangle$	$\cdots$	$\langle \mathfrak{T}_{2k}(\sqcap), \mathfrak{I}_{2k}(\sqcap), \mathfrak{F}_{2k}(\sqcap) \rangle$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$\mathsf{C}_n^{\mathbf{S}}$	$\langle \mathfrak{T}_{n1}(\sqcap), \mathfrak{I}_{n1}(\sqcap), \mathfrak{F}_{n1}(\sqcap) \rangle$	$\langle \mathfrak{T}_{n2}(\sqcap), \mathfrak{I}_{n2}(\sqcap), \mathfrak{F}_{n2}(\sqcap) \rangle$	$\cdots$	$\langle \mathfrak{T}_{nk}(\sqcap), \mathfrak{I}_{nk}(\sqcap), \mathfrak{F}_{nk}(\sqcap) \rangle$

Step 3. Convert NRSs to SVNSs

Using the neutrosophic power mean operator defined in Eq. (1), we convert NRSs in $\mathcal{M}^1$ to SVNSs and produce in $\mathcal{M}^3$.

$\mathcal{M}^3$: **Candidates versus attributes SVNSs D-Matrix**

	$\mathsf{C}_1^{\mathbf{S}}$	$\mathsf{C}_2^{\mathbf{S}}$	$\cdots$	$\mathsf{C}_n^{\mathbf{S}}$
$\mathbb{A}_1^{\mathbf{S}}$	$\langle \mathfrak{T}_{11}(\sqcap), \mathfrak{I}_{11}(\sqcap), \mathfrak{F}_{11}(\sqcap) \rangle$	$\langle \mathfrak{T}_{12}(\sqcap), \mathfrak{I}_{12}(\sqcap), \mathfrak{F}_{12}(\sqcap) \rangle$	$\cdots$	$\langle \mathfrak{T}_{1n}(\sqcap), \mathfrak{I}_{1n}(\sqcap), \mathfrak{F}_{1n}(\sqcap) \rangle$
$\mathbb{A}_2^{\mathbf{S}}$	$\langle \mathfrak{T}_{21}(\sqcap), \mathfrak{I}_{21}(\sqcap), \mathfrak{F}_{21}(\sqcap) \rangle$	$\langle \mathfrak{T}_{22}(\sqcap), \mathfrak{I}_{22}(\sqcap), \mathfrak{F}_{22}(\sqcap) \rangle$	$\cdots$	$\langle \mathfrak{T}_{2n}(\sqcap), \mathfrak{I}_{2n}(\sqcap), \mathfrak{F}_{2n}(\sqcap) \rangle$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$\mathbb{A}_m^{\mathbf{S}}$	$\langle \mathfrak{T}_{m1}(\sqcap), \mathfrak{I}_{m1}(\sqcap), \mathfrak{F}_{m1}(\sqcap) \rangle$	$\langle \mathfrak{T}_{m2}(\sqcap), \mathfrak{I}_{m2}(\sqcap), \mathfrak{F}_{m2}(\sqcap) \rangle$	$\cdots$	$\langle \mathfrak{T}_{mn}(\sqcap), \mathfrak{I}_{mn}(\sqcap), \mathfrak{F}_{mn}(\sqcap) \rangle$

Step 4. Calculation of CE-measure values

We calculate the CE-measure values between$\mathcal{M}^2$ and $\mathcal{M}^3$.

Step 5. Order the alternatives according to the results of the evaluation.

The available alternatives are prioritized according to the ascending values of the CE-measure. A lower CE-value indicates a more preferable available alternative.

Step 6. End

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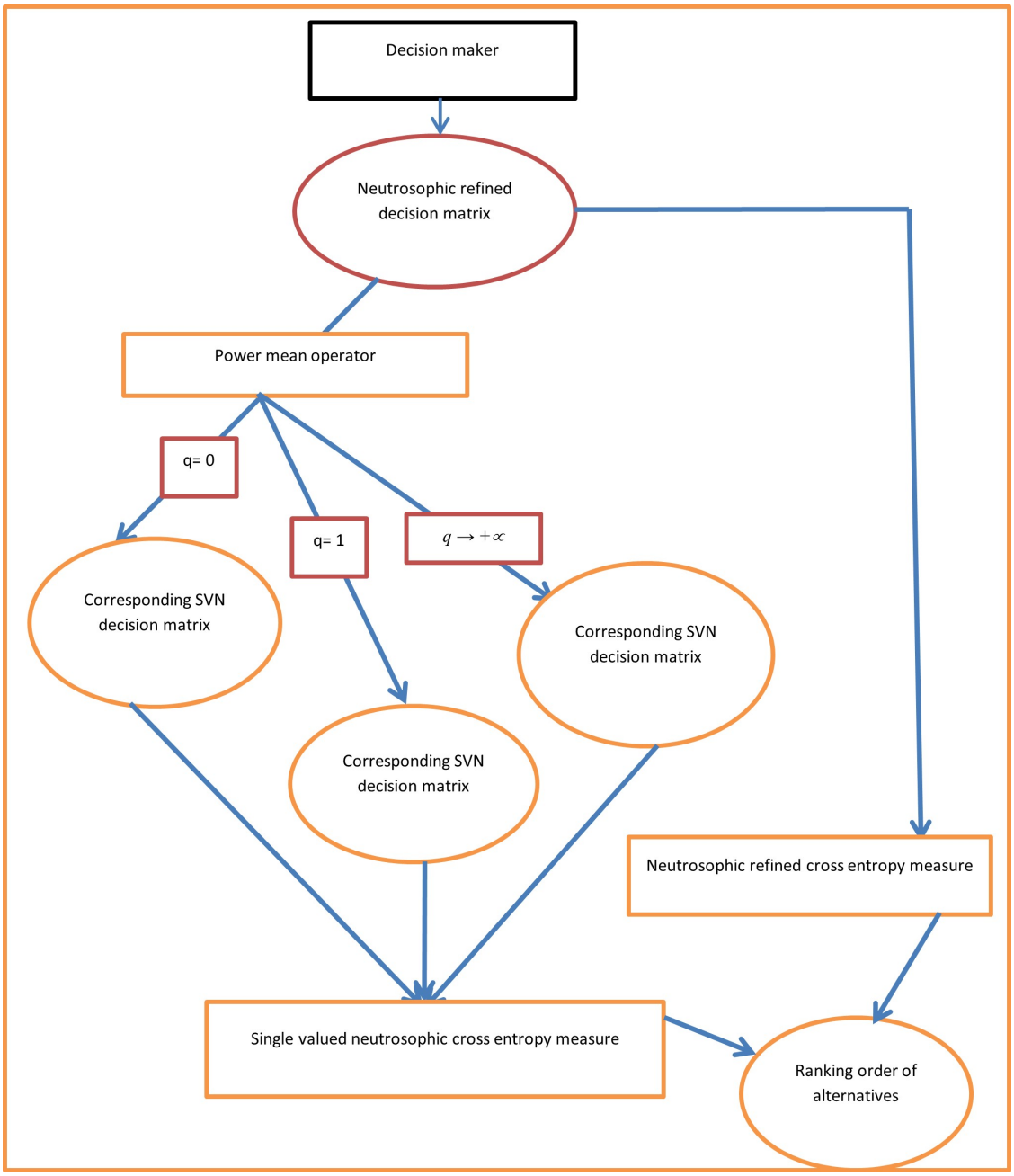

Figure 1. Proposed decision-making procedure.

6. Example: Selection of an Appropriate Educational Stream

Drawing inspiration from [38], we consider an MADM problem related to selecting an appropriate educational stream for higher studies. Choosing the right path after completing the 10+2 level is often a challenge for students and their families. In this example, we determine the preference ranking of various streams for each student using a set of evaluative attributes (see Table 3).

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Table 3. Description of aspirant students, their required attributes, and higher educational available streams.

Candidates (Symbol)	**Description**
$\mathbb{A}_1^{\mathbf{S}}$ $\mathbb{A}_2^{\mathbf{S}}$ $\mathbb{A}_3^{\mathbf{S}}$	These are three students of Rahara Ramakrishna Mission Boys' Home (India). They have completed their basic course (10+2 level).
Attributes (Symbol)	**Description**
$\mathsf{C}_1^{\mathbf{S}}$	Proficiency in both English and Bengali languages
$\mathsf{C}_2^{\mathbf{S}}$	Depth in Mathematics and basic computers knowledge
$\mathsf{C}_3^{\mathbf{S}}$	In-depth understanding of scientific disciplines.
$\mathsf{C}_4^{\mathbf{S}}$	Concentration
$\mathsf{C}_5^{\mathbf{S}}$	Laborious
Educational Streams (Symbol)	**Description**
$\mathbf{B}_1^{\mathbf{s}}$	Mathematics Honors
$\mathbf{B}_2^{\mathbf{s}}$	Physics Honors
$\mathbf{B}_3^{\mathbf{s}}$	Engineering
$\mathbf{B}_4^{\mathbf{s}}$	ComputerScience
$\mathbf{B}_5^{\mathbf{s}}$	Biochemistry

Step 1. Formulation of a D-Matrix between candidates and evaluation attributes.

The relationship between aspirant students and their corresponding attributes is modeled using NRSs, based on three mutually independent evaluations provided by a decision maker. The resulting triplet of relational values for each student is represented in $\mathcal{M}^4$.

$\mathcal{M}^4$: Candidates versus attributes neutrosophic refined D-Matrix

	$\mathsf{C}_1^{\mathbf{S}}$	$\mathsf{C}_2^{\mathbf{S}}$	$\mathsf{C}_3^{\mathbf{S}}$	$\mathsf{C}_4^{\mathbf{S}}$	C_5
	$\langle 0.7, 0.2, 0.2\rangle$	$\langle 0.6, 0.2, 0.4\rangle$	$\langle 0.6, 0.3, 0.2\rangle$	$\langle 0.7, 0.2, 0.4\rangle$	$\langle 0.7, 0.6, 0.4\rangle$
$\mathbb{A}_1^{\mathbf{S}}$	$\langle 0.6, 0.3, 0.3\rangle$	$\langle 0.5, 0.2, 0.4\rangle$	$\langle 0.6, 0.5, 0.2\rangle$	$\langle 0.7, 0.4, 0.4\rangle$	$\langle 0.6, 0.4, 0.5\rangle$
	$\langle 0.6, 0.3, 0.1\rangle$	$\langle 0.5, 0.1, 0.2\rangle$	$\langle 0.7, 0.3, 0.4\rangle$	$\langle 0.6, 0.3, 0.3\rangle$	$\langle 0.6, 0.5, 0.4\rangle$
	$\langle 0.8, 0.4, 0.4\rangle$	$\langle 0.5, 0.5, 0.2\rangle$	$\langle 0.8, 0.2, 0.2\rangle$	$\langle 0.6, 0.6, 0.2\rangle$	$\langle 0.6, 0.6, 0.4\rangle$
$\mathbb{A}_2^{\mathbf{S}}$	$\langle 0.7, 0.6, 0.4\rangle$	$\langle 0.5, 0.4, 0.1\rangle$	$\langle 0.8, 0.2, 0.5\rangle$	$\langle 0.6, 0.7, 0.5\rangle$	$\langle 0.7, 0.6, 0.4\rangle$
	$\langle 0.8, 0.6, 0.4\rangle$	$\langle 0.6, 0.6, 0.3\rangle$	$\langle 0.8, 0.4, 0.2\rangle$	$\langle 0.8, 0.5, 0.4\rangle$	$\langle 0.8, 0.6, 0.3\rangle$
	$\langle 0.7, 0.2, 0.2\rangle$	$\langle 0.6, 0.1, 0.1\rangle$	$\langle 0.6, 0.3, 0.4\rangle$	$\langle 0.6, 0.5, 0.1\rangle$	$\langle 0.6, 0.5, 0.5\rangle$
$\mathbb{A}_3^{\mathbf{S}}$	$\langle 0.6, 0.4, 0.1\rangle$	$\langle 0.6, 0.4, 0.2\rangle$	$\langle 0.6, 0.5, 0.5\rangle$	$\langle 0.6, 0.5, 0.5\rangle$	$\langle 0.8, 0.5, 0.3\rangle$
	$\langle 0.5, 0.3, 0.3\rangle$	$\langle 0.6, 0.1, 0.3\rangle$	$\langle 0.7, 0.4, 0.6\rangle$	$\langle 0.6, 0.6, 0.3\rangle$	$\langle 0.8, 0.3, 0.4\rangle$

Step 2. Formation of attributes versus alternatives D-Matrix

The evaluation scores of the available alternatives $\mathbf{B}_t^{\mathbf{s}}$ $(t = 1, 2, \ldots, k)$ with reference to the required attributes C_i $(i = 1, 2, \ldots, n)$ are represented using SVNSs. These SVNS-based assessments are arranged in the alternatives-versus-attributes decision matrix, denoted by $\mathcal{M}^5$.

$\mathcal{M}^5$: Attributes versus alternatives SVNSs D-Matrix

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	C_1^S	C_2^S	C_3^S	C_4^S	C_5^S
B_1^s	$\langle 0.5, 0.2, 0.4\rangle$	$\langle 0.9, 0.2, 0.2\rangle$	$\langle 0.6, 0.4, 0.3\rangle$	$\langle 0.6, 0.2, 0.4\rangle$	$\langle 0.7, 0.2, 0.4\rangle$
B_2^s	$\langle 0.4, 0.5, 0.4\rangle$	$\langle 0.8, 0.4, 0.2\rangle$	$\langle 0.8, 0.2, 0.4\rangle$	$\langle 0.6, 0.1, 0.3\rangle$	$\langle 0.6, 0.4, 0.3\rangle$
B_3^s	$\langle 0.4, 0.4, 0.4\rangle$	$\langle 0.7, 0.3, 0.4\rangle$	$\langle 0.6, 0.3, 0.4\rangle$	$\langle 0.6, 0.5, 0.3\rangle$	$\langle 0.6, 0.5, 0.4\rangle$
B_4^s	$\langle 0.5, 0.3, 0.3\rangle$	$\langle 0.8, 0.4, 0.2\rangle$	$\langle 0.8, 0.4, 0.2\rangle$	$\langle 0.6, 0.5, 0.4\rangle$	$\langle 0.7, 0.6, 0.4\rangle$
B_5^s	$\langle 0.5, 0.4, 0.4\rangle$	$\langle 0.7, 0.4, 0.2\rangle$	$\langle 0.8, 0.4, 0.3\rangle$	$\langle 0.6, 0.5, 0.4\rangle$	$\langle 0.7, 0.2, 0.2\rangle$

Step 3. Convert NRNs to SVNN

Using the neutrosophic power mean operator defined in Eq. (1) and different values of q, we obtain the following matrices.

When $q = 0$, the single valued decision matrix corresponding to the candidates versus attributes is represented in $\mathcal{M}^6$.

$\mathcal{M}^6$: Candidates versus attributes converted SVNS D-Matrix

	C_1^S	C_2^S	C_3^S	C_4^S	C_5^S
$\mathbb{A}_1^S$	$\langle 0.6, 0.3, 0.2\rangle$	$\langle 0.5, 0.2, 0.2\rangle$	$\langle 0.6, 0.4, 0.2\rangle$	$\langle 0.7, 0.3, 0.4\rangle$	$\langle 0.6, 0.5, 0.4\rangle$
$\mathbb{A}_2^S$	$\langle 0.8, 0.5, 0.4\rangle$	$\langle 0.5, 0.5, 0.2\rangle$	$\langle 0.8, 0.2, 0.3\rangle$	$\langle 0.6, 0.6, 0.3\rangle$	$\langle 0.7, 0.6, 0.4\rangle$
$\mathbb{A}_3^S$	$\langle 0.6, 0.3, 0.2\rangle$	$\langle 0.6, 0.1, 0.2\rangle$	$\langle 0.6, 0.5, 0.5\rangle$	$\langle 0.7, 0.5, 0.2\rangle$	$\langle 0.7, 0.4, 0.4\rangle$

When $q = 1$

$\mathcal{M}^7$: Candidates versus attributes converted SVNS D-Matrix

	C_1^S	C_2^S	C_3^S	C_4^S	C_5^S
$\mathbb{A}_1^S$	$\langle 0.6, 0.3, 0.2\rangle$	$\langle 0.5, 0.2, 0.3\rangle$	$\langle 0.6, 0.4, 0.3\rangle$	$\langle 0.7, 0.3, 0.4\rangle$	$\langle 0.6, 0.5, 0.4\rangle$
$\mathbb{A}_2^S$	$\langle 0.8, 0.5, 0.4\rangle$	$\langle 0.5, 0.5, 0.2\rangle$	$\langle 0.8, 0.2, 0.4\rangle$	$\langle 0.6, 0.6, 0.4\rangle$	$\langle 0.7, 0.6, 0.4\rangle$
$\mathbb{A}_3^S$	$\langle 0.6, 0.3, 0.2\rangle$	$\langle 0.6, 0.1, 0.3\rangle$	$\langle 0.6, 0.5, 0.5\rangle$	$\langle 0.7, 0.5, 0.3\rangle$	$\langle 0.7, 0.4, 0.4\rangle$

When $q \to \alpha$

$\mathcal{M}^8$: Candidates versus attributes converted SVNS D-Matrix

	C_1^S	C_2^S	C_3^S	C_4^S	C_5^S
$\mathbb{A}_1^S$	$\langle 0.7, 0.3, 0.3\rangle$	$\langle 0.6, 0.2, 0.4\rangle$	$\langle 0.7, 0.5, 0.4\rangle$	$\langle 0.7, 0.4, 0.4\rangle$	$\langle 0.7, 0.6, 0.5\rangle$
$\mathbb{A}_2^S$	$\langle 0.8, 0.6, 0.4\rangle$	$\langle 0.6, 0.6, 0.3\rangle$	$\langle 0.8, 0.2, 0.5\rangle$	$\langle 0.6, 0.7, 0.5\rangle$	$\langle 0.8, 0.6, 0.4\rangle$
$\mathbb{A}_3^S$	$\langle 0.7, 0.4, 0.3\rangle$	$\langle 0.6, 0.2, 0.4\rangle$	$\langle 0.7, 0.6, 0.6\rangle$	$\langle 0.8, 0.6, 0.5\rangle$	$\langle 0.8, 0.5, 0.5\rangle$

Step 4. Calculation of cross entropy measure values

Using Eq. (9), we compute the cross-entropy values between $\mathcal{M}^5$ and $\mathcal{M}^6$ (see matrix $\mathcal{M}^9$); $\mathcal{M}^5$ and $\mathcal{M}^7$ (see matrix $\mathcal{M}^{10}$); and $\mathcal{M}^5$ and $\mathcal{M}^8$ (see matrix $\mathcal{M}^{11}$). The resulting cross-entropy values are presented in the following table:

$\mathcal{M}^9$: Cross-entropy values between $\mathcal{M}^5$ and $\mathcal{M}^6$

Candidates	Mathematics Honors (B_1^s)	Physics Honors (B_2^s)	Engineering (B_3^s)	Computer Science (B_4^s)	Biochemistry (B_5^s)
$\mathbb{A}_1^S$	1.34	2.12	1.52	**1.27**	1.77
$\mathbb{A}_2^S$	2.26	1.54	1.59	**1.39**	1.46
$\mathbb{A}_3^S$	1.67	2.22	1.51	**1.50**	1.78

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$\mathcal{M}^{10}$: Cross entropy values between $\mathcal{M}^5$ and $\mathcal{M}^7$

Candidates	Mathematics Honors (B_1^s)	Physics Honors (B_2^s)	Engineering (B_3^s)	Computer Science (B_4^s)	Biochemistry (B_5^s)
$\mathbb{A}_1^S$	1.41	2.06	**1.18**	1.42	1.62
$\mathbb{A}_2^S$	2.28	1.57	1.59	**1.32**	1.48
$\mathbb{A}_3^S$	1.68	2.22	**1.34**	1.58	1.78

$\mathcal{M}^{11}$: Cross entropy values between $\mathcal{M}^5$ and $\mathcal{M}^8$

Candidates	Mathematics Honors (B_1^s)	Physics Honors (B_2^s)	Engineering (B_3^s)	Computer Science (B_4^s)	Biochemistry (B_5^s)
$\mathbb{A}_1^S$	1.76	2.24	1.41	**1.32**	1.85
$\mathbb{A}_2^S$	3.01	2.12	**1.70**	1.86	2.31
$\mathbb{A}_3^S$	2.47	2.85	**1.69**	2.02	2.21

Applying Eq. (10), the CE-measures between $\mathcal{M}^4$ and $\mathcal{M}^5$ are computed and reported in $\mathcal{M}^{12}$.

$\mathcal{M}^{12}$: Cross entropy values between $\mathcal{M}^4$ and $\mathcal{M}^5$

Candidates	B_1^s: Mathematics Honors	B_2^s: Physics Honors	B_3^s: Engineering	B_4^s: Computer Science	B_5^s: Biochemistry
$\mathbb{A}_1^S$	1.50	2.12	**1.46**	1.53	2.02
$\mathbb{A}_2^S$	2.56	1.95	1.70	**1.53**	1.65
$\mathbb{A}_3^S$	1.96	2.42	**1.54**	1.79	1.91

Step 5. Ordering of the alternatives according to the results of the evaluation.

The preference order of the available alternatives, based on the increasing values of the cross-entropy values, is given as follows (See Table 4).

Table 4. Ranking order of alternatives.

Values of q	Candidates	Priority ranking order of streams
$q = 0$	$\mathbb{A}_1^S$	$\mathbf{B}_4^s \succ \mathbf{B}_1^s \succ \mathbf{B}_3^s \succ \mathbf{B}_5^s \succ \mathbf{B}_2^s$
	$\mathbb{A}_2^S$	$\mathbf{B}_4^s \succ \mathbf{B}_5^s \succ \mathbf{B}_2^s \succ \mathbf{B}_3^s \succ \mathbf{B}_1^s$
	$\mathbb{A}_3^S$	$\mathbf{B}_4^s \succ \mathbf{B}_3^s \succ \mathbf{B}_1^s \succ \mathbf{B}_5^s \succ \mathbf{B}_2^s$
$q = 1$	$\mathbb{A}_1^S$	$\mathbf{B}_3^s \succ \mathbf{B}_1^s \succ \mathbf{B}_4^s \succ \mathbf{B}_5^s \succ \mathbf{B}_2^s$
	$\mathbb{A}_2^S$	$\mathbf{B}_4^s \succ \mathbf{B}_5^s \succ \mathbf{B}_2^s \succ \mathbf{B}_3^s \succ \mathbf{B}_1^s$
	$\mathbb{A}_3^S$	$\mathbf{B}_3^s \succ \mathbf{B}_4^s \succ \mathbf{B}_1^s \succ \mathbf{B}_5^s \succ \mathbf{B}_2^s$
$q \to \infty$	$\mathbb{A}_1^S$	$\mathbf{B}_4^s \succ \mathbf{B}_3^s \succ \mathbf{B}_1^s \succ \mathbf{B}_5^s \succ \mathbf{B}_2^s$
	$\mathbb{A}_2^S$	$\mathbf{B}_3^s \succ \mathbf{B}_4^s \succ \mathbf{B}_2^s \succ \mathbf{B}_5^s \succ \mathbf{B}_1^s$
	$\mathbb{A}_3^S$	$\mathbf{B}_3^s \succ \mathbf{B}_4^s \succ \mathbf{B}_5^s \succ \mathbf{B}_1^s \succ \mathbf{B}_2^s$
Using Eq. (10)	$\mathbb{A}_1^S$	$\mathbf{B}_3^s \succ \mathbf{B}_1^s \succ \mathbf{B}_4^s \succ \mathbf{B}_5^s \succ \mathbf{B}_2^s$
	$\mathbb{A}_2^S$	$\mathbf{B}_4^s \succ \mathbf{B}_5^s \succ \mathbf{B}_3^s \succ \mathbf{B}_2^s \succ \mathbf{B}_1^s$
	$\mathbb{A}_3^S$	$\mathbf{B}_3^s \succ \mathbf{B}_4^s \succ \mathbf{B}_5^s \succ \mathbf{B}_1^s \succ \mathbf{B}_2^s$

Table 4 shows that, when $q = 0$,

1. Computer Science ($\mathbf{B}_4^s$) is the best stream for candidates $\mathbb{A}_1^S$, $\mathbb{A}_2^S$, and $\mathbb{A}_3^S$.

When $q = 1$,

1. Engineering ($\mathbf{B}_3^s$) is the best stream for candidate $\mathbb{A}_1^S$.
2. Computer Science ($\mathbf{B}_4^s$) is the best stream for the candidate $\mathbb{A}_2^S$.
3. Engineering ($\mathbf{B}_3^s$) is the best stream for candidate $\mathbb{A}_3^S$.

When $q \to \infty$,

1. Computer Science ($\mathbf{B}_4^s$) is the best stream for candidate $\mathbb{A}_1^S$.
2. Engineering ($\mathbf{B}_3^s$) is the best stream for candidate $\mathbb{A}_2^S$.

Using Eq. (11)

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1. Engineering ($\mathbf{B}_3^{\mathbf{s}}$) is the best stream for candidate $\mathbb{A}_1^{\mathbf{S}}$.
2. Computer Science ($\mathbf{B}_4^{\mathbf{s}}$) is the best stream for candidate $\mathbb{A}_2^{\mathbf{S}}$.
3. Engineering ($\mathbf{B}_3^{\mathbf{s}}$) is the best stream for candidate $\mathbb{A}_3^{\mathbf{S}}$.

Hence, we conclude that the best suitable stream for each candidate changes with the different values of q and two different cross-entropy measures. Therefore, the new operator emphasizes the judgment of uncertainty and provides more sensible and consistent results for MADM problems.

7. Conclusions

Intelligent decision-making techniques-play a crucial role when choices must be made under complex and uncertain conditions. MADM approaches, in particular, have extensive applications in advanced decision analysis. In this work, we introduce a new power mean operator within the neutrosophic refined set (NRS) framework to address MADM problems involving refined neutrosophic information.

To begin with, we formulate the power mean operator in the NRS environment and establish its fundamental properties. This operator is used to transform NRSs into single-valued neutrosophic sets (SVNSs). Additionally, we develop two types of cross-entropy measures in the NRS setting: one based on the SVNS cross-entropy measure after transformation, and the other defined directly on NRSs without requiring conversion. Using these measures, an MADM method is constructed.

An educational decision-making case study is introduced to highlight the effectiveness of the proposed approach. Owing to the significance of the introduced power mean operator, several promising research directions are anticipated. The operator is expected to be applicable in aggregating decision information and designing decision-making procedures in areas such as teacher selection [7], school choice [61], and investment planning [16], among others.

Author Contributions: All authors contributed equally.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The abbreviations employed in this manuscript are listed below:

MADA	Multi-attribute decision making
CEM	Cross Entropy Measure
DM	Decision-making
SVNS	Single valued neutrosophic set
RNSs	eutrosophic refined sets

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